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**Module One – 1-5 Problem Set**

**Problem one:**  
  
(a) Every patient was given the medication or the placebo or both.

Logical Expression: ∀x (D(x) ∨ P(x))

Negation: ¬∀x (D(x) ∨ P(x))

Applying De Morgan’s law: ∃x ¬(D(x) ∨ P(x))

Further Application: ∃x (¬D(x) ∧ ¬P(x))

English Translation: There is at least one patient who was given neither the medication nor the placebo.

(b) Every patient who took the placebo had migraines

Logical Expression: ∀x (P(x) → M(x))

Conditional Identity Applied: ∀x (¬P(x) ∨ M(x))

Negation: ¬∀x (¬P(x) ∨ M(x))

Applying De Morgan’s law: ∃x ¬(¬P(x) ∨ M(x))

Further Application: ∃x (P(x) ∧ ¬M(x))

English Translation: There is at least one patient who took the placebo and did not have migraines.

(c) There is a patient who had migraines and was given the placebo.

Logical Expression: ∃x (M(x) ∧ P(x))

Negation: ¬∃x (M(x) ∧ P(x))

Applying De Morgan’s law: ∀x ¬(M(x) ∧ P(x))

Further Application: ∀x (¬M(x) ∨ ¬P(x))

English Translation: Every patient either did not have migraines or was not given the placebo or both.

**Problem Two:**  
  
(a) ¬∀x (P(x) ∧ ¬Q(x)) ≡ ∃x (¬P(x) ∨ Q(x))

Beginning with ¬∀x (P(x) ∧ ¬Q(x)). The negation of a universal quantifier is equivalent to the existence quantifier with a negated statement: ¬∀x (P(x) ∧ ¬Q(x)) ≡ ∃x ¬(P(x) ∧ ¬Q(x))

Then I applied De Morgan’s law to move the negation inside the parentheses: ∃x ¬(P(x) ∧ ¬Q(x)) ≡ ∃x (¬P(x) ∨ ¬¬Q(x))

Then I simplified by removing the double negation: ∃x (¬P(x) ∨ ¬¬Q(x)) ≡ ∃x (¬P(x) ∨ Q(x))

Thus, the equivalence is proven.

(b) ¬∀x (¬P(x) → Q(x)) ≡ ∃x (¬P(x) ∧ ¬Q(x))

Beginning this problem with ¬∀x (¬P(x) → Q(x)). The implication can be rewritten using the identity ( p → q ≡ ¬p ∨ q ): ¬∀x (¬P(x) → Q(x)) ≡ ¬∀x (P(x) ∨ Q(x))

The negation of a universal quantifier is the existence quantifier with the negated statement: ¬∀x (P(x) ∨ Q(x)) ≡ ∃x ¬(P(x) ∨ Q(x))

Then, applying De Morgan’s law to move the negation inside: ∃x ¬(P(x) ∨ Q(x)) ≡ ∃x (¬P(x) ∧ ¬Q(x))

Thus, the equivalence is proven.

(c) ¬∃x (¬P(x) ∨ (Q(x) ∧ ¬R(x))) ≡ ∀x (P(x) ∧ (¬Q(x) ∨ R(x)))

Start with ¬∃x (¬P(x) ∨ (Q(x) ∧ ¬R(x))). The negation of an existential quantifier is a universal quantifier with a negated statement: ¬∃x (¬P(x) ∨ (Q(x) ∧ ¬R(x))) ≡ ∀x ¬(¬P(x) ∨ (Q(x) ∧ ¬R(x)))

Then, applying De Morgan’s law to the negation of the disjunction and the conjunction inside: ∀x ¬(¬P(x) ∨ (Q(x) ∧ ¬R(x))) ≡ ∀x (P(x) ∧ ¬(Q(x) ∧ ¬R(x)))

Again, I Applied De Morgan’s law again to the negation of the conjunction: ∀x (P(x) ∧ ¬(Q(x) ∧ ¬R(x))) ≡ ∀x (P(x) ∧ (¬Q(x) ∨ ¬¬R(x)))

Then I simplified by removing the double negation: ∀x (P(x) ∧ (¬Q(x) ∨ ¬¬R(x))) ≡ ∀x (P(x) ∧ (¬Q(x) ∨ R(x)))

Thus, the equivalence is proven.

**Problem Three:**  
  
(a) ∀x∀y (x ≠ y) → M(x, y)  
Since M(2,3) is false, the statement doesn’t hold for all pairs x and y where x is not equal to y. Thus the statement is false.  
  
(b) ∀x∃y ¬M(x, y)  
Since this statement must hold for all x, and there is no person y to who person 1 who has not sent an email, thus the statement is also false.  
  
(c) ∃x∀y M(x, y)  
Since there is at least one x ( x = 1) for which M(x,y) is true for all y; thus the statement is true.

**Problem Four:**  
  
(a) The reciprocal of every positive number less than one is greater than one.  
This can be expressed as a universal statement. The predicate in this involves a number being positive, less than one, and the reciprocal being > 1.  
Logical expression: ∀x ((0 < x < 1) → (1/x > 1))  
  
(b) There is no smallest number.  
To say there is no smallest number means that for every number, there is another number that is smaller than it.  
Logical expression: ∀x ∃y (y < x)  
  
(c) Every number other than 0 has a multiplicative inverse.  
This is applied for every number except zero; there exists another number which when multiplied by it gives one.  
Logical expression: ∀x (x ≠ 0 → ∃y (x \* y = 1))

**Problem Five:**  
  
(a) Write an element from the set A x B x C.  
  
An element would be a 3-tuple where the first element comes from set A, the second from set B, and the third would come from set C.  
Example element: tall, foam, non-fat  
  
(b) Write an element from the set B x A x C.  
  
This would also be a 3-tuple where the first element comes from set B, the second from set A, and the third from set C.  
Example element: no-foam, grande, whole  
  
(c) Write the set B x C using roster notation.  
  
B x C would consist of all possible ordered pairs where the first element is from set B and the second is from set C.  
B x C in roster notation: {(foam,non−fat),(foam,whole),(no−foam,non−fat),(no−foam,whole)}